Light-Front Holography:
a New Approach to Nonperturbative QCD

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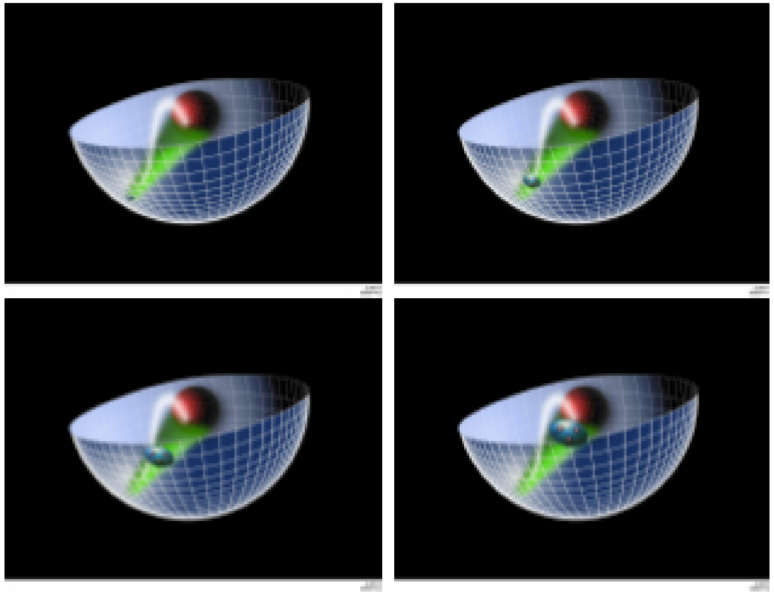

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Recent Review: GdT and S. J. Brodsky, arXiv:1203.4025 [hep-ph]

Gauge/Gravity Correspondence and QCD

- Recent developments inspired by the AdS/CFT correspondence [Maldacena (1998)] between gravity in AdS space and conformal field theories in physical space-time provide physical insights into non-perturbative QCD dynamics

- Description of strongly coupled gauge theory using a dual gravity description in a higher dimensional space (holographic)

- Isomorphism of $SO(4,2)$ group of conformal transformations with generators $P^\mu, M^{\mu\nu}, K^\mu, D,$ with the group of isometries of AdS$_5$, a space of maximal symmetry, negative curvature and a four-dim boundary: Minkowski space

- Mapping of AdS gravity theory to boundary QFT quantized at fixed light-front time gives a precise relation between holographic wave functions in AdS space and the light-front wavefunctions describing the internal structure of hadrons

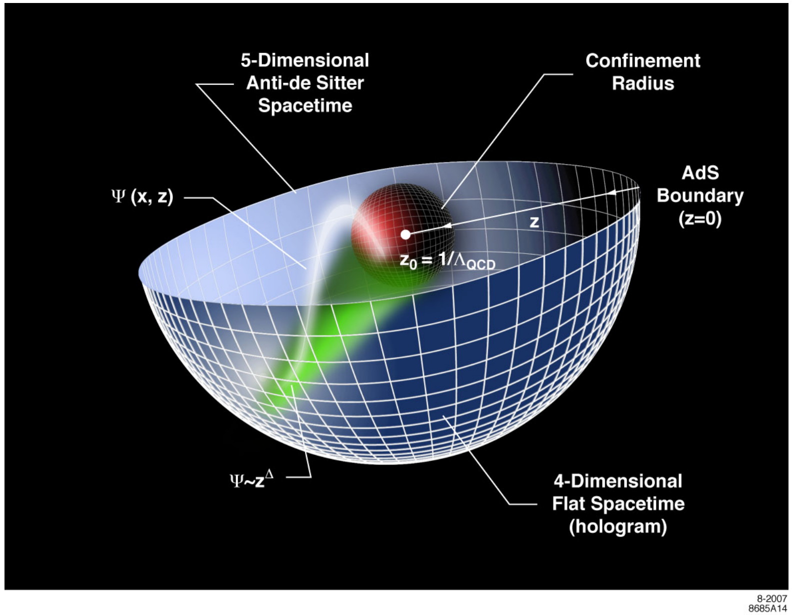
- The gauge/gravity duality leads to a simple analytical frame-independent nonperturbative semiclassical approximation to the full light-front QCD Hamiltonian: “Light-Front Holography”

- AdS$_5$ metric: $$\underbrace{ds^2}_{L_{\rm AdS}} = \frac{R^2}{z^2}\Big(\underbrace{\eta_{\mu\nu}dx^\mu dx^\nu}_{L_{\rm Minkowski}} - dz^2\Big)$$

- A distance $L_{\rm AdS}$ shrinks by a warp factor z/R as observed in Minkowski space $(dz = 0)$:

$$L_{\rm Minkowski} \sim \frac{z}{R}\, L_{\rm AdS}$$

- Since the AdS metric is invariant under a dilatation of all coordinates $x^\mu \to \lambda x^\mu$, $z \to \lambda z$, the variable z acts like a scaling variable in Minkowski space



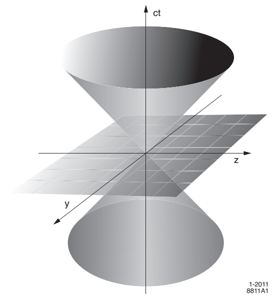
- Short distances $x_\mu x^\mu \to 0$ maps to UV conformal AdS$_5$ boundary $z \to 0$

- Large confinement dimensions $x_\mu x^\mu \sim 1/\Lambda^2_{\rm QCD}$ maps to large IR region of AdS$_5$, $z \sim 1/\Lambda_{\rm QCD}$, thus there is a maximum separation of quarks and a maximum value of z

Forms of Relativistic Dynamics

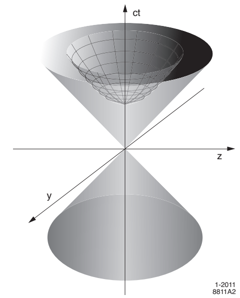
- Different possibilities to parametrize space-time [Dirac (1949)]
- Parametrizations differ by the hypersurface on which the initial conditions are specified. Each evolve with different "times" and has its own Hamiltonian, but should give the same physical results
- Forms of Relativistic Dynamics: dynamical *vs.* kinematical generators [Dirac (1949)]
- *Instant form*: hypersurface defined by $t = 0$, the familiar one

$$H, \, \mathbf{K} \;\; \text{dynamical}, \qquad \mathbf{L}, \, \mathbf{P} \;\; \text{kinematical}$$



- *Point form*: hypersurface is an hyperboloid

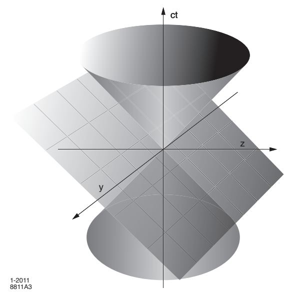
$$P^{\mu} \;\; \text{dynamical}, \qquad M^{\mu\nu} \;\; \text{kinematical}$$



- *Front form*: hypersurface is tangent to the light cone at $\tau = t + z/c = 0$

$$P^{-}, \, L^{x}, \, L^{y} \;\; \text{dynamical}, \qquad P^{+}, \, \mathbf{P}_{\perp}, \, L^{z}, \, \mathbf{K} \;\; \text{kinematical}$$

$$P^{\pm} = P^{0} \pm P^{3}$$



Light-Front Quantization of QCD

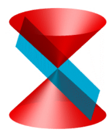
- Express the hadron four-momentum generator $P = (P^+, P^-, \mathbf{P}_\perp)$ in terms of dynamical fields

$$
\begin{aligned}
P^- &= \frac{1}{2}\int dx^- d^2\mathbf{x}_\perp \overline{\psi}_+ \, \gamma^+ \frac{(i\nabla_\perp)^2 + m^2}{i\partial^+}\psi_+ + (\text{interactions}),\\
P^+ &= \int dx^- d^2\mathbf{x}_\perp \overline{\psi}_+ \gamma^+ i\partial^+ \psi_+,\\
\mathbf{P}_\perp &= \frac{1}{2}\int dx^- d^2\mathbf{x}_\perp \overline{\psi}_+ \gamma^+ i\nabla_\perp \psi_+,
\end{aligned}
$$

where the integrals are over the null plane $\tau = x^+ = x^0 + x^3$

- Construct LF Lorentz invariant Hamiltonian equation for the relativistic bound state

$$
\boxed{H_{LF}|\psi(P)\rangle = \mathcal{M}^2|\psi(P)\rangle}
$$



with $H_{LF} \equiv P_\mu P^\mu = P^- P^+ - \mathbf{P}_\perp^2$

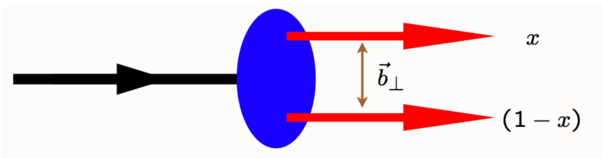
Semiclassical Approximation to QCD in the Light Front

[GdT and S. J. Brodsky, PRL **102**, 081601 (2009)]

- Compute $\mathcal{M}^2$ from hadronic matrix element $\langle\psi(P')|P_\mu P^\mu|\psi(P)\rangle = \mathcal{M}^2\langle\psi(P')|\psi(P)\rangle$
- State $|\psi(P)\rangle$ is expanded in multi-particle Fock states $|n\rangle$ of the free LF Hamiltonian

$$|\psi\rangle = \sum_n \psi_n |n\rangle$$

- Relevant variable for a two-parton system $\zeta^2 = x(1-x)\mathbf{b}_\perp^2$



- To first approximation LF dynamics depend only on the invariant variable ζ, and hadronic properties are encoded in the hadronic mode $\phi(\zeta)$ from

$$\psi(x,\zeta,\varphi) = e^{iL^z\varphi} X(x) \frac{\phi(\zeta)}{\sqrt{2\pi\zeta}}$$

factor angular φ, longitudinal $X(x)$ and transverse mode $\phi(\zeta)$ (P^+, $\mathbf{P}_\perp$ and J_z commute with P^-)

- Ultra relativistic limit $m_q \to 0$ longitudinal modes $X(x)$ decouple $\quad (L = |L^z|)$

$$\mathcal{M}^2 = \int d\zeta \, \phi^*(\zeta) \sqrt{\zeta} \left(-\frac{d^2}{d\zeta^2} - \frac{1}{\zeta}\frac{d}{d\zeta} + \frac{L^2}{\zeta^2} \right) \frac{\phi(\zeta)}{\sqrt{\zeta}} + \int d\zeta \, \phi^*(\zeta) \, U(\zeta) \, \phi(\zeta)$$

where the confining forces from the interaction terms are summed up in the effective potential $U(\zeta)$

- LF eigenvalue equation $P_\mu P^\mu |\phi\rangle = \mathcal{M}^2 |\phi\rangle$ is a LF wave equation for ϕ

$$\boxed{\left(\underbrace{-\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2}}_{kinetic\ energy\ of\ partons} + \underbrace{U(\zeta)}_{confinement} \right) \phi(\zeta) = \mathcal{M}^2 \phi(\zeta)}$$

- Effective relativistic and frame-independent LF Schrödinger equation: U is instantaneous in LF time
- Eigenmodes $\phi(\zeta)$ determine the hadronic mass spectrum and represent the probability amplitude to find n-massless partons at transverse impact separation ζ within the hadron at equal LF time
- Semiclassical approximation to LF QCD does not account for particle creation and absorption

Light-Front Holographic Mapping of Wave Equations

Higher Spin Modes in AdS Space

- Spin-J in AdS represented by totally symmetric rank J tensor field $\Phi_{M_1 \cdots M_J}$
- Action for spin-J field in AdS$_{d+1}$ in presence of dilaton background $\varphi(z)$ $\left(x^M = (x^\mu, z)\right)$

$$S = \frac{1}{2} \int d^d x \, dz \, \sqrt{g} \, e^{\varphi(z)} \Big(g^{NN'} g^{M_1 M_1'} \cdots g^{M_J M_J'} D_N \Phi_{M_1 \cdots M_J} D_{N'} \Phi_{M_1' \cdots M_J'} - \mu^2 g^{M_1 M_1'} \cdots g^{M_J M_J'} \Phi_{M_1 \cdots M_J} \Phi_{M_1' \cdots M_J'} + \cdots \Big)$$

where D_M is the covariant derivative which includes parallel transport

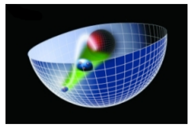
- Physical hadron has plane-wave and polarization indices along $3+1$ physical coordinates

$$\Phi_P(x, z)_{\mu_1 \cdots \mu_J} = e^{-iP \cdot x} \Phi(z)_{\mu_1 \cdots \mu_J}, \quad \Phi_{z \mu_2 \cdots \mu_J} = \cdots = \Phi_{\mu_1 \mu_2 \cdots z} = 0$$

with four-momentum P_μ and invariant hadronic mass $P_\mu P^\mu = M^2$

- Find AdS wave equation for spin J-mode $\Phi_J = \Phi_{\mu_1 \cdots \mu_J}$ and all indices along 3+1

$$\left[- \frac{z^{d-1-2J}}{e^{\varphi(z)}} \partial_z \left(\frac{e^{\varphi}(z)}{z^{d-1-2J}} \partial_z \right) + \left(\frac{\mu R}{z} \right)^2 \right] \Phi_J(z) = \mathcal{M}^2 \Phi_J(z)$$



Dual QCD Light-Front Wave Equation

$$z \Leftrightarrow \zeta, \quad \Phi_P(z) \Leftrightarrow |\psi(P)\rangle$$

[GdT and S. J. Brodsky, PRL **102**, 081601 (2009)]

- Upon substitution $z \rightarrow \zeta$ and $\phi_J(\zeta) \sim \zeta^{-3/2+J} e^{\varphi(z)/2}\, \Phi_J(\zeta)$ in AdS WE

$$\left[-\frac{z^{d-1-2J}}{e^{\varphi(z)}} \partial_z \left(\frac{e^{\varphi}(z)}{z^{d-1-2J}} \partial_z\right) + \left(\frac{\mu R}{z}\right)^2\right] \Phi_J(z) = \mathcal{M}^2 \Phi_J(z)$$

find LFWE $\quad (d = 4)$

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right) \phi_J(\zeta) = M^2 \phi_J(\zeta)$$

with

$$U(\zeta) = \frac{1}{2}\varphi''(z) + \frac{1}{4}\varphi'(z)^2 + \frac{2J-3}{2z}\varphi'(z)$$

and $(\mu R)^2 = -(2-J)^2 + L^2$

- AdS Breitenlohner-Freedman bound $(\mu R)^2 \geq -4$ equivalent to LF QM stability condition $L^2 \geq 0$
- Scaling dimension τ of AdS mode Φ_J is $\tau = 2 + L$ in agreement with twist scaling dimension of a two parton bound state in QCD and determined by QM stability condition

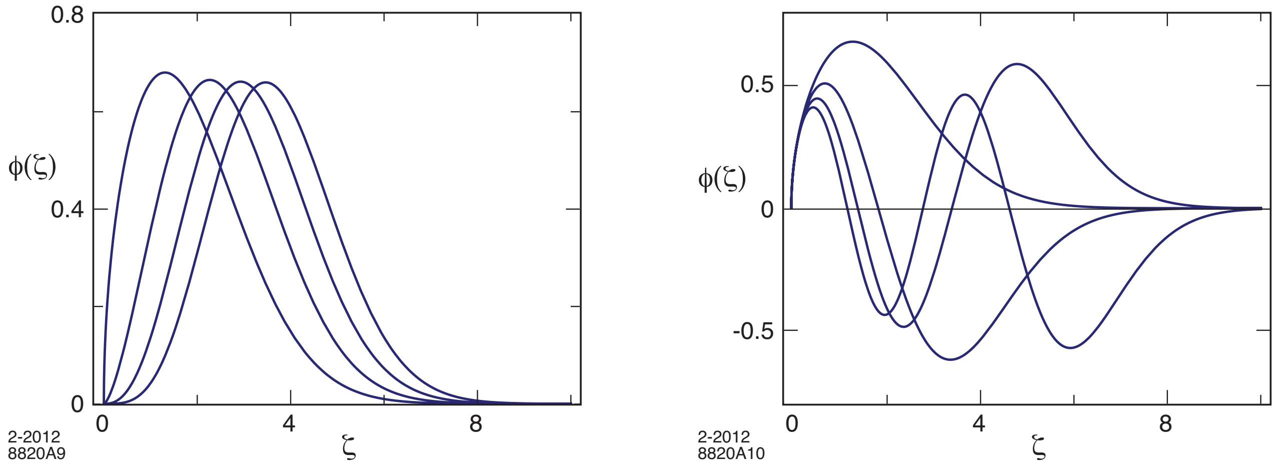
Bosonic Modes and Meson Spectrum

Soft wall model: linear Regge trajectories [Karch, Katz, Son and Stephanov (2006)]

- Dilaton profile $\varphi(z) = +\kappa^2 z^2$
- Effective potential: $U(z) = \kappa^4 \zeta^2 + 2\kappa^2(J-1)$
- Normalized eigenfunctions $\langle \phi | \phi \rangle = \int d\zeta \, |\phi(z)^2| = 1$

$$\phi_{n,L}(\zeta) = \kappa^{1+L} \sqrt{\frac{2n!}{(n+L)!}} \, \zeta^{1/2+L} e^{-\kappa^2 \zeta^2/2} L_n^L(\kappa^2 \zeta^2)$$

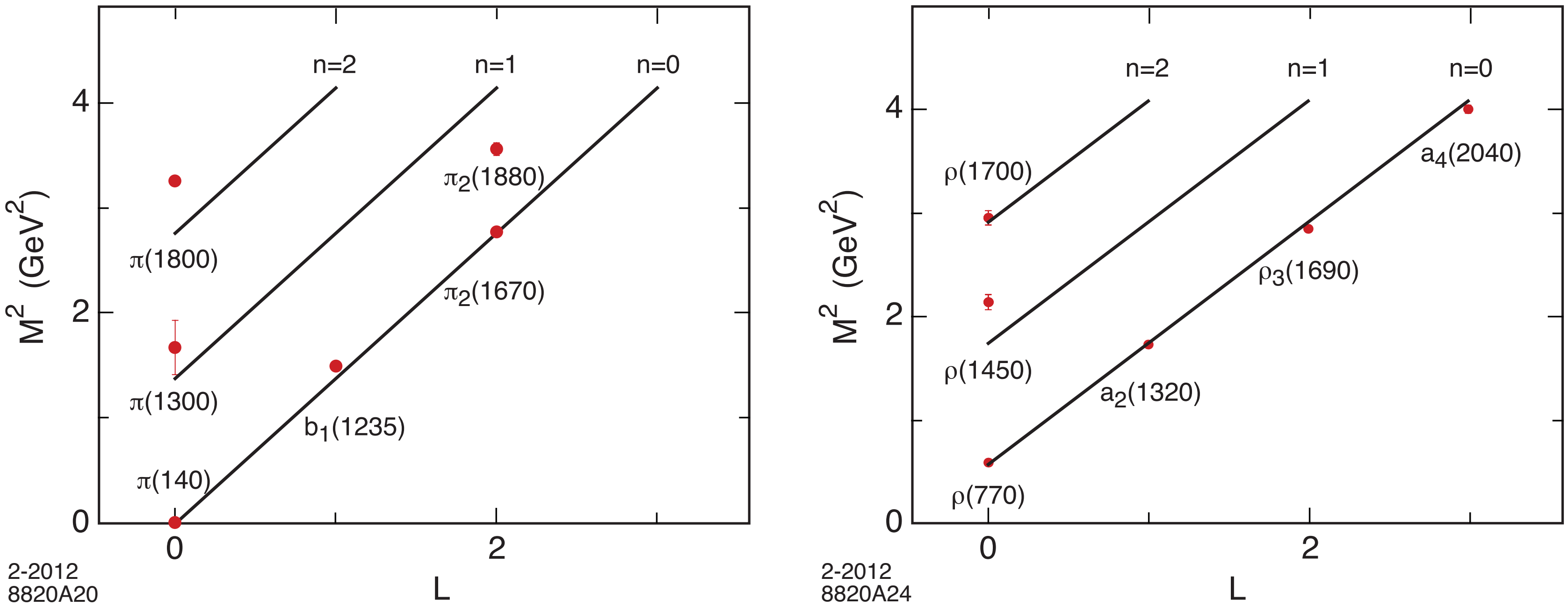
- Eigenvalues $\mathcal{M}^2_{n,L,S} = 4\kappa^2 \left(n + \frac{J+L}{2}\right)$



LFWFs $\phi_{n,L}(\zeta)$ in physical space-time: L) orbital modes and R) radial modes

- $J = L + S$, $I = 1$ meson families $\mathcal{M}^2_{n,L,S} = 4\kappa^2\left(n + L + S/2\right)$

$4\kappa^2$ for $\Delta n = 1$
$4\kappa^2$ for $\Delta L = 1$
$2\kappa^2$ for $\Delta S = 1$

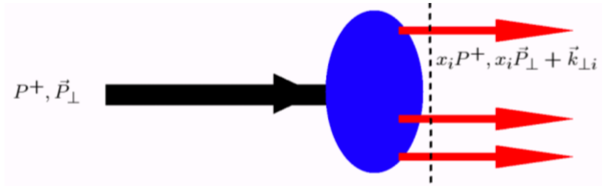


I=1 orbital and radial excitations for the π ($\kappa = 0.59$ GeV) and the ρ-meson families ($\kappa = 0.54$ GeV)

- Triplet splitting for the $I = 1$, $L = 1$, $J = 0, 1, 2$, vector meson a-states

$$\mathcal{M}_{a_2(1320)} > \mathcal{M}_{a_1(1260)} > \mathcal{M}_{a_0(980)}$$

Multi-Partonic States and Cluster Decomposition



- Proton state $|\psi(P)\rangle$

$$|\psi\rangle = \sum_n \psi_n |n\rangle, \qquad |n\rangle = \{\, |uud\rangle, |uudg\rangle, |uud\bar{q}q\rangle, \ \cdots \}$$

- Extension of LF holography to arbitrary n follows from the x-weighted definition of the transverse impact variable of the $n-1$ spectator system

  [S. J. Brodsky and GdT, PRL **96**, 201601 (2006)]

$$\zeta = \sqrt{\frac{x}{1-x}} \Big| \sum_{j=1}^{n-1} x_j \mathbf{b}_{\perp j} \Big|$$

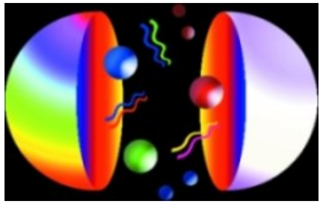
  where $x = x_n$ is the longitudinal momentum fraction of the active quark

- Same multiplicity of states for mesons and baryons !

Fermionic Modes and Baryon Spectrum

[LF Hard wall model: GdT and S. J. Brodsky, PRL **94**, 201601 (2005)]

[LF Soft wall model: GdT and S. J. Brodsky, (2005), arXiv:1001.5193]



From Nick Evans

- Excitation spectrum of nucleon represents formidable challenge to LQCD due to enormous computational complexity beyond ground state configuration
- LF Holographic nucleon modes

$$\psi_+(\zeta)_{n,L} = \kappa^{2+L}\sqrt{\frac{2n!}{(n+L)!}}\zeta^{3/2+L}e^{-\kappa^2\zeta^2/2}L_n^{L+1}\left(\kappa^2\zeta^2\right)$$

$$\psi_-(\zeta)_{n,L} = \kappa^{3+L}\frac{1}{\sqrt{n+L+2}}\sqrt{\frac{2n!}{(n+L)!}}\zeta^{5/2+L}e^{-\kappa^2\zeta^2/2}L_n^{L+2}\left(\kappa^2\zeta^2\right)$$

- Normalization

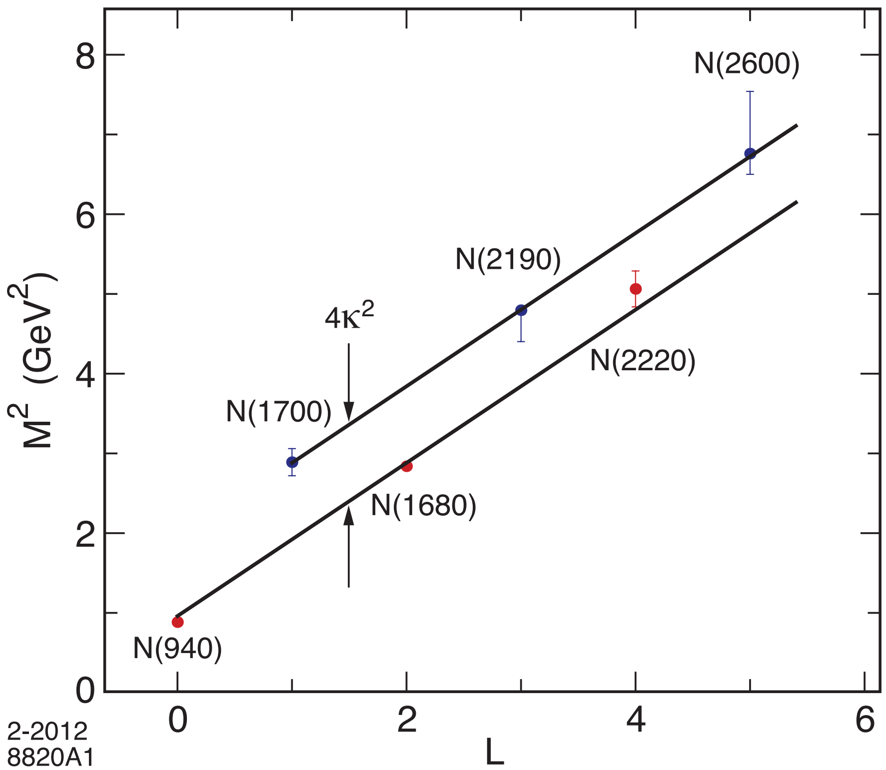
$$\int d\zeta\,\psi_+^2(\zeta) = \int d\zeta\,\psi_-^2(\zeta) = 1$$

- Eigenvalues

$$\mathcal{M}_{n,L,S}^{2(+)} = 4\kappa^2\left(n+L+S/2+3/4\right)$$

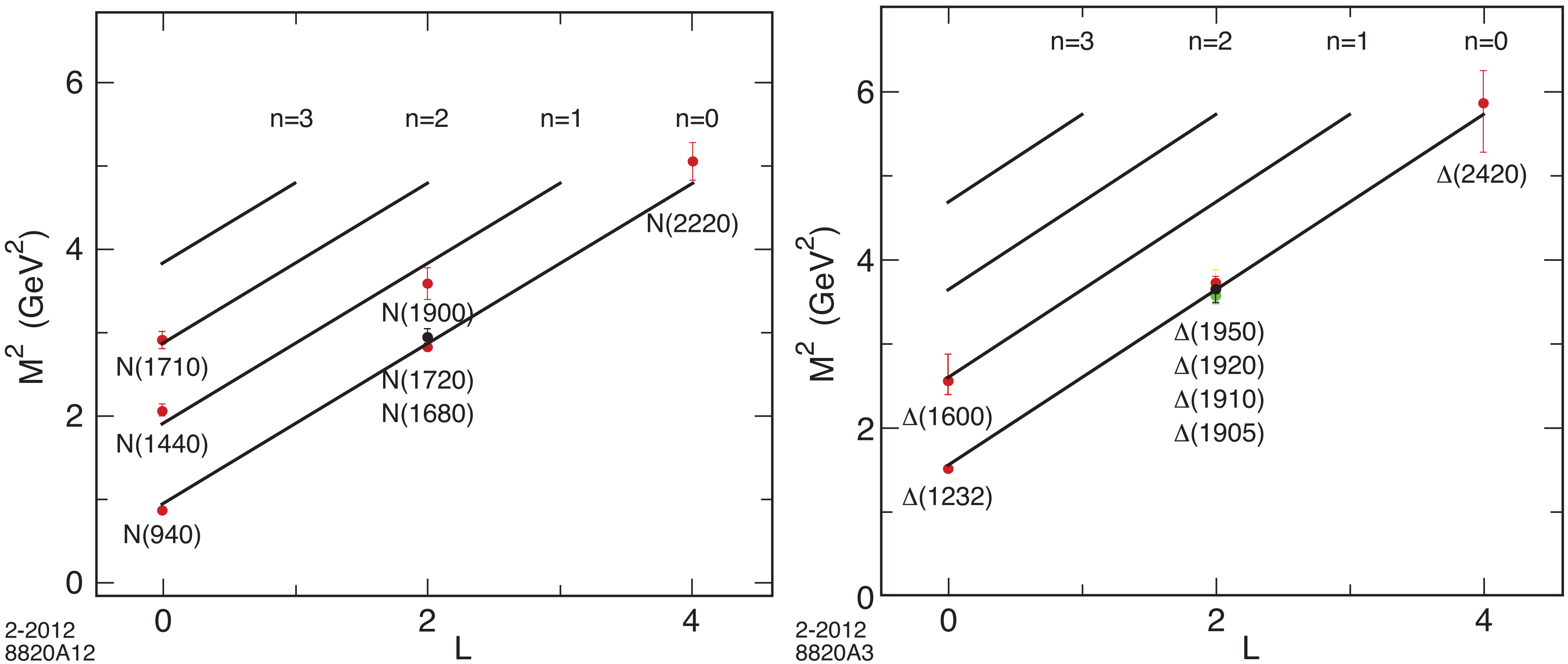
$$\mathcal{M}_{n,L,S}^{2(-)} = 4\kappa^2\left(n+L+S/2+5/4\right)$$

- Gap scale $4\kappa^2$ determines trajectory slope and spectrum gap between plus-parity spin-$\frac{1}{2}$ and minus-parity spin-$\frac{3}{2}$ nucleon families for the branch solutions $L+1 = \mu R - 1/2$ and $L+1 = \mu R + 1/2$



Plus-minus nucleon spectrum gap for $\kappa = 0.49$ GeV

$4\kappa^2$ for $\Delta n = 1$
$4\kappa^2$ for $\Delta L = 1$
$2\kappa^2$ for $\Delta S = 1$



Orbital and radial excitations for positive parity N and Δ baryon families ($\kappa = 0.49 - 0.51$ GeV)

Same results for the Δ spectrum: H. Forkel, M. Beyer and T. Frederico, JHEP **0707**, 077 (2007)

Light-Front Holographic Mapping of Current Matrix Elements

[S. J. Brodsky and GdT, PRL **96**, 201601 (2006)]

- EM transition matrix element in QCD: local coupling to pointlike constituents

$$\langle \psi(P') | J^\mu | \psi(P) \rangle = (P + P')^\mu F(Q^2)$$

where $Q = P' - P$ and $J^\mu = e_q \overline{q} \gamma^\mu q$

- EM hadronic matrix element in AdS space from coupling of external EM field propagating in AdS with extended mode $\Phi(x, z)$

$$\int d^4x \, dz \, \sqrt{g} \, A^M(x, z) \Phi^*_{P'}(x, z) \overleftrightarrow{\partial}_M \Phi_P(x, z)$$

$$\sim (2\pi)^4 \delta^4 \left(P' - P\right) \epsilon_\mu \left(P + P'\right)^\mu F(Q^2)$$

- How to recover hard pointlike scattering at large Q out of soft collision of extended objects?

[Polchinski and Strassler (2002)]

- Mapping of J^+ elements at fixed light-front time: $\Phi_P(z) \Leftrightarrow |\psi(P)\rangle$

- Compare with electromagnetic FF in LF QCD for arbitrary Q. Expressions can be matched only if LFWF is factorized

$$\psi(x, \zeta, \varphi) = e^{iM\varphi} X(x) \frac{\phi(\zeta)}{\sqrt{2\pi\zeta}}$$

- Find

$$X(x) = \sqrt{x(1-x)}, \qquad \phi(\zeta) = \left(\frac{\zeta}{R}\right)^{-3/2} \Phi(\zeta), \qquad z \to \zeta$$

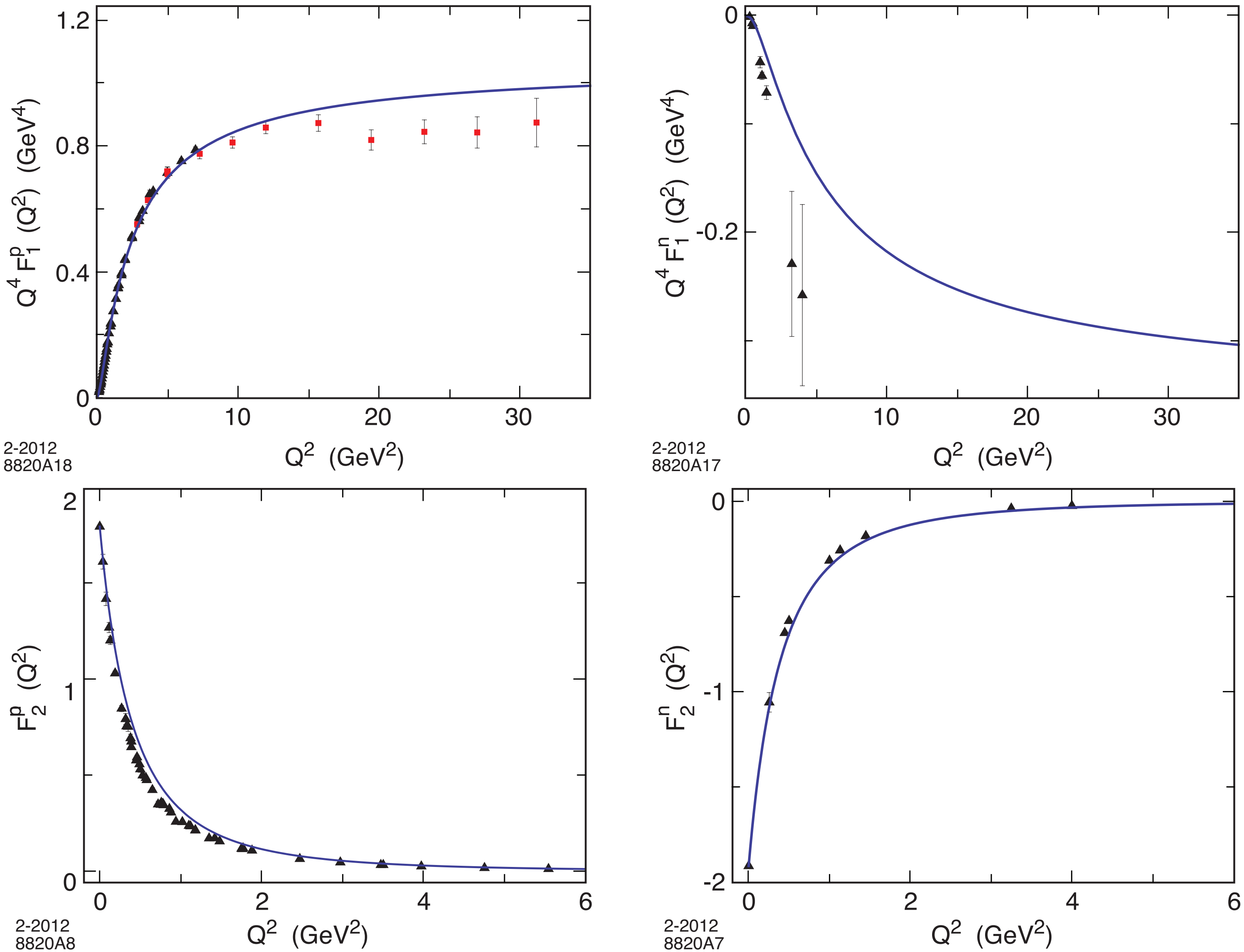
- Form factor in soft-wall model expressed as $\tau - 1$ product of poles along vector radial trajectory (twist $\tau = N + L$) [Brodsky and GdT, Phys.Rev. D77 (2008) 056007]

$$F_\tau(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1 + \frac{Q^2}{\mathcal{M}_{\rho'}^2}\right) \cdots \left(1 + \frac{Q^2}{\mathcal{M}_{\rho^{\tau-2}}^2}\right)}$$

where ${\mathcal{M}_{\rho_n}}^2 \to 4\kappa^2(n + 1/2)$

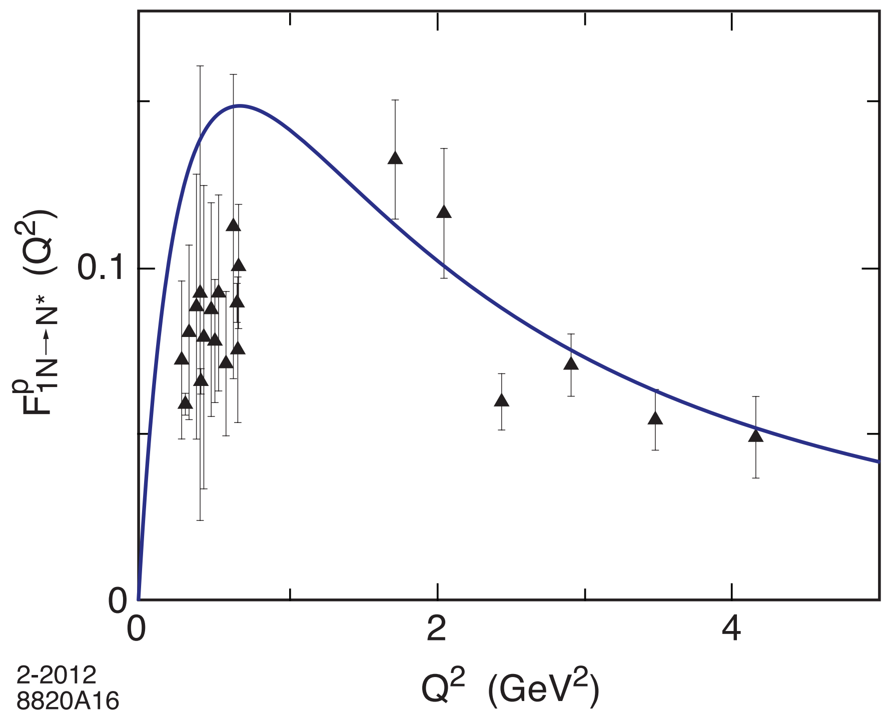
- Analytical form $F(Q^2)$ incorporates correct scaling from constituents and mass gap from confinement

Nucleon Elastic Form Factors



Nucleon Transition Form Factors

$$F^{p}_{1\,N\to N^*}(Q^2) = \frac{\sqrt{2}}{3}\,\frac{\frac{Q^2}{\mathcal{M}_\rho^2}}{\left(1+\frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1+\frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)\left(1+\frac{Q^2}{\mathcal{M}_{\rho''}^2}\right)}.$$



Proton transition form factor to the first radial excited state

Flavor Decomposition of Elastic Nucleon Form Factors

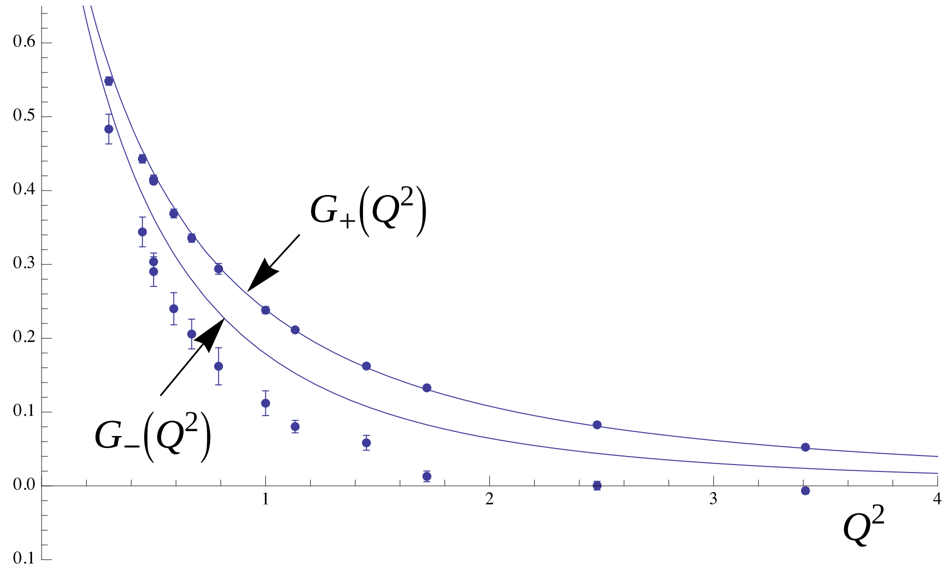
G. D. Cates *et al.* Phys. Rev. Lett. **106**, 252003 (2011)

- Proton SU(6) WF: $F^p_{u,1} = \frac{5}{3}G_+ + \frac{1}{3}G_-,\qquad F^p_{d,1} = \frac{1}{3}G_+ + \frac{2}{3}G_-$
- Neutron SU(6) WF: $F^n_{u,1} = \frac{1}{3}G_+ + \frac{2}{3}G_-,\qquad F^n_{d,1} = \frac{5}{3}G_+ + \frac{1}{3}G_-$

$$G_+(Q^2) = \frac{1}{\left(1+\frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1+\frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)}$$

and

$$G_-(Q^2) = \frac{1}{\left(1+\frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1+\frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)\left(1+\frac{Q^2}{\mathcal{M}_{\rho''}^2}\right)}$$



PRELIMINARY